Volume 2, No.7, July, 2014

Journal of Global Research in Mathematical Archives

ISSN 2320 – 5822

REVIEW PAPER

Available online at http://www.jgrma.com

STRATEGIES STREAMLINES PATTERN FOR THE SECOND GRADE FLUID

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Abstract: In this paper, the general solution is found for the partial differential equations describing the steady plane motion of incompressible second grade fluid in the presence of unknown body force when vorticity function satisfies the equation. The general solution of the partial differential equation is obtained using the method of variation of parameters and the method of separation. The exact solutions are found for the flow equations in which four arbitrary functions are involved. Stream functions are found for the steady plane flow equation and the corresponding streamline patterns are drawn. The effects of second grade parameter in the stream function and streamlines patterns are studied and the effect of porosity parameter also discussed for the steady plane motion. The results are reported for conclusion.

Keywords: Second Grade Fluid, Porosity Parameter, Stream Line, Stream Function

1 INTRODUCTION

The second grade fluids are the common non-Newtonian viscoelastic fluids in industrial fields, such as polymer solutions. second grade fluids have been a famous topic of research because of their diverse use in many industrial processes. Various complex fluids such as oils, polymer melts, different types of drilling mud's and clay coatings and many emulsions are included in the category of second grade fluids.

Second grade fluids in porous media exhibit a nonlinear behavior different from that of Newtonian fluids in porous media. The rheological effects of second grade fluids through porous media occur in a board range of engineering applications, e.g., transport processes in chemical industry, storage of nuclear waste material, discoveries of the flow of oil in petroleum reservoirs, and food processing. Interest in the study of second grade fluids has been mainly motivated by their importance in most of the problems arising from engineering practice and chemical industry. Amongst them, the second grade fluids of differential type have received special attention from the research scholars.

The problem of finding exact solutions of governing equations of second grade fluids flow present insuperable mathematical difficulties due to the fact that these are nonlinear. However, researchers using various methods and approaches have determined exact solutions to flow equations.

Recently, Nameem (2012) presented some new exact solutions to equations governing the plane steady motion of incompressible second grade fluids in the presence of unknown body force for prescribed vorticity distribution. And this work is extended for the study of the effects of porosity for the steady plane motion.

2 MATHEMATICAL FORMULATION OF THE PROBLEM

The basic equations describing the flow of a viscous incompressible second grade fluid in the presence of body force, neglecting thermal effects are

The basic equations describing the flow of a viscous incompressible second grade fluid in the presence of body force, neglecting thermal effects are

$$div V = 0 \quad (3.1.1)$$

$$div T + \rho f = \rho V \quad (3.1.2)$$

And the constitutive equation for the Cauchy stress T is

$$T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1{}^2. \quad (3.1.3)$$

Here V is the velocity vector, ρ is the density, f is the body force per unit mass, p is the dynamic pressure, μ is the coefficient of dynamic viscosity and α_1 & α_2 are the normal-stress moduli. Rivlin-Ericksen tensors A_1 and A_2 are given by

$$A_1 = (\nabla V) + (\nabla V)^T, \qquad (3.1.4)$$
$$A_2 = A_1 + (\nabla V)^T A_1 + A_1(\nabla V).$$

And h is the generalized function for steady plane flows defined by

$$h = p + \frac{1}{2}\rho(u^2 + v^2) - \alpha_1(u\nabla^2 u + v\nabla^2 v) - \frac{1}{4}(3\alpha_1 + 2\alpha_2)\left(4u_x{}^2 + 4v_y{}^2 + 2(u_y + v_x)^2\right) \qquad (3.1.5)$$

and differentiate the above equations with respect to x & y to obtain the flow equations

$$h_x = -\rho v\omega - \mu\omega_y + \alpha_1 v\nabla^2\omega + \rho f_1, \qquad (3.1.6)$$
$$h_y = -\rho u\omega + \mu\omega_x + \alpha_1 u\nabla^2\omega + \rho f_2, \qquad (3.1.7)$$

where f_1, f_2 are the force components.

The equations of continuity of an incompressible fluid given below

$$u_x + v_y = 0, \qquad (3.1.8)$$
$$\omega = v_x - u_y \qquad (3.1.9)$$

and ω is the vorticity function.

Equation (3.1.8) implies the existence of the stream function ψ such that

$$u = \psi_y, v = -\psi_x. \qquad (3.1.10)$$

Inserting equation (3.1.10) into equations (3.1.6-3.1.9), to obtain the solution for

$$\omega = -\nabla^2\psi, \qquad (3.1.11)$$
$$h_x = -\rho\psi_x\omega - \mu\omega_y + \alpha_1\psi_x\nabla^2\omega + F_1 \qquad (3.1.12)$$
$$h_y = -\rho\psi_y\omega + \mu\omega_x + \alpha_1\psi_y\nabla^2\omega + F_2 \qquad (3.1.13)$$
$$\text{where} \quad F_1 = \rho f_1, F_2 = \rho f_2. \qquad (3.1.14)$$

Once a solution of equations (3.1.11-3.1.13) is determined, the velocity components and the pressure p are determined employing equations (3.1.10) and (3.1.5), respectively.

2.1 General Solution of the Existence Stream Function

The general solution of equations (3.1.11-3.1.13) for the flows characterized by the equation

$$\alpha_1\nabla^2\omega - \rho\omega = Z \qquad (3.2.1)$$
$$\text{Where} \quad Z = G(x) + M(y) \qquad (3.2.2)$$

in equation (3.2.2), the functions G(x) and M(y) are arbitrary and substituting the value of ω is

$$\omega = A(x) + B(y) \qquad (3.2.3)$$

equation (3.2.1) divides by α_1

$$\nabla^2\omega - \frac{\rho}{\alpha_1}\omega = \frac{z}{\alpha_1} \qquad (3.2.4)$$

In this equation (3.2.1), to obtain the solution substituting

$\frac{\rho}{\alpha_1} = \lambda$ and the value of ω in (3.2.4)

$$A''(x) + B''(y) - \frac{\rho}{\alpha_1}(A(x) + B(y)) = \frac{G(x) + M(y)}{\alpha_1}$$
$$A''(x) + B''(y) - \lambda(A(x) + B(y)) = \frac{G(x) + M(y)}{\alpha_1} \qquad (3.2.5)$$

And add and subtract a separation constant λ_0 in (3.2.5)

$$A''(x) + B''(y) - \lambda(A(x) + B(y)) = \frac{G(x)}{\alpha_1} + \lambda_0 - \lambda_0 + \frac{M(y)}{\alpha_1} \qquad (3.2.6)$$

Separate the variable x and y in equation (3.2.6)

$$A''(x) - \lambda A(x) = \lambda_0 + \frac{G(x)}{\alpha_1} \quad (3.2.7)$$

$$B''(y) - \lambda B(y) = -\lambda_0 + \frac{M(y)}{\alpha_1} \quad (3.2.8)$$

where $\lambda = \frac{\rho}{\alpha_1}, \lambda_0$ is separation constant

Now consider applying method of variation of parameters on equations (3.2.7)

$$A''(x) - \lambda A(x) = \lambda_0 + \frac{G(x)}{\alpha_1}$$

Consider auxiliary equation of equation (3.2.7)

$$m^2 - \lambda = 0$$

$$\Rightarrow m = \pm\sqrt{\lambda}$$

$$A(x) = C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x} \quad (3.2.9)$$

using variation parameter method formula to find particular integral of (3.2.7). The formula for particular integral is

$$y_p = y_1 \int \frac{-y_2 g(x)}{\omega(y_1, y_2)} dx + y_2 \int \frac{y_1 g(x)}{\omega(y_1, y_2)} dx \quad (3.2.10)$$

$$\omega(y_1, y_2) = y_1 y_2' - y_1' y_2 \quad (3.2.11)$$

and y_1, y_2 are auxiliary solutions of (3.2.9)

where $y_1 = e^{\sqrt{\lambda}x}$ $y_2 = e^{-\sqrt{\lambda}x}$ its get from (3.2.9) and differentiate with respect to x the value of y_1, y_2 *and* to get substitute this in equations (3.2.11) to obtain

$$\omega(y_1, y_2) =$$

\

$$e^{\sqrt{\lambda}x}\left(-\sqrt{\lambda}e^{-\sqrt{\lambda}x}\right) - \sqrt{\lambda}e^{\sqrt{\lambda}x}e^{-\sqrt{\lambda}x}$$

$$= -\sqrt{\lambda} - \sqrt{\lambda} = -2\sqrt{\lambda}$$

$$\omega(y_1, y_2) = -2\sqrt{\lambda} \quad (3.2.12)$$

and to know that $y = \lambda_0 + \frac{G(x)}{\alpha_1}$ from (3.2.7)

Now consider the particular integral and substitute the values of equations (3.2.11) and (3.2.7) in (3.2.10)

$$y_p = y_1 \int \frac{-y_2 g(x)}{\omega(y_1, y_2)} dx + y_2 \int \frac{y_1 g(x)}{\omega(y_1, y_2)} dx$$

$$y_p = e^{\sqrt{\lambda}x} \int \frac{-e^{-\sqrt{\lambda}x}}{-2\sqrt{\lambda}}\left(\lambda_0 + \frac{G(x)}{\alpha_1}\right)dx +$$

$$e^{-\sqrt{\lambda}x} \int \frac{e^{-\sqrt{\lambda}x}}{-2\sqrt{\lambda}}\left(\lambda_0 + \frac{G(x)}{\alpha_1}\right)dx$$

$$y_p = \frac{e^{\sqrt{\lambda}x}}{2\sqrt{\lambda}} \int \left(\lambda_0 + \frac{G(x)}{\alpha_1}\right)e^{-\sqrt{\lambda}x}dx -$$

$$\frac{e^{-\sqrt{\lambda}x}}{2\sqrt{\lambda}} \int \left(\lambda_0 + \frac{G(x)}{\alpha_1}\right)e^{\sqrt{\lambda}x}dx \quad (3.2.13)$$

$y = C.I + P.I$ is (add the equations (3.2.9) + (3.2.13))

$$A(x) = C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x} + \frac{e^{\sqrt{\lambda}x}}{2\sqrt{\lambda}} \int \left(\lambda_0 + \frac{G(x)}{\alpha_1}\right)e^{-\sqrt{\lambda}x}dx - \frac{e^{-\sqrt{\lambda}x}}{2\sqrt{\lambda}} \int \left(\lambda_0 + \frac{G(x)}{\alpha_1}\right)e^{\sqrt{\lambda}x}dx \quad (3.2.14)$$

Similarly applying method of variation of parameters on equations (3.2.8) is

$$B''(y) - \lambda B(y) = -\lambda_0 + \frac{m(y)}{\alpha_1}$$

The auxiliary equation of (3.2.8) is

$$m^2 - \lambda = 0$$
$$m = \pm\sqrt{\lambda}$$
$$B(y) = C_3 e^{\sqrt{\lambda}y} + C_4 e^{-\sqrt{\lambda}y} \qquad (3.2.15)$$

Now consider the particular integral equations of (3.2.10), where $y_1 = e^{\sqrt{\lambda}y}; y_2 = e^{-\sqrt{\lambda}y}$ the answer obtain from (3.2.15) and differentiate with respect to y the value of y_1, y_2 $y_1' = \sqrt{\lambda}e^{\sqrt{\lambda}y}; y_2' = -\sqrt{\lambda}e^{-\sqrt{\lambda}y}$ substitute this in equations (3.2.11) to get

$$\omega(y_1, y_2) =$$
$$e^{\sqrt{\lambda}y}\left(-\sqrt{\lambda}e^{-\sqrt{\lambda}y}\right) - \sqrt{\lambda}e^{\sqrt{\lambda}y}e^{-\sqrt{\lambda}y}$$
$$= -\sqrt{\lambda} - \sqrt{\lambda} = -2\sqrt{\lambda}$$
$$\omega(y_1, y_2) = -2\sqrt{\lambda} \qquad (3.2.16)$$

$$y = -\lambda_0 + \frac{m(y)}{\alpha_1} \text{ its obtain from (3.2.8)}$$

$$y_p = e^{\sqrt{\lambda}y}\int \frac{-e^{-\sqrt{\lambda}y}}{-2\sqrt{\lambda}}\left(-\lambda_0 + \frac{m(y)}{\alpha_1}\right)dy + e^{-\sqrt{\lambda}y}\int \frac{e^{-\sqrt{\lambda}y}}{-2\sqrt{\lambda}}\left(-\lambda_0 + \frac{m(y)}{\alpha_1}\right)dy$$

$$y_p = \frac{e^{\sqrt{\lambda}y}}{2\sqrt{\lambda}}\int\left(-\lambda_0 + \frac{m(y)}{\alpha_1}\right)e^{-\sqrt{\lambda}y}dy - \frac{e^{-\sqrt{\lambda}y}}{2\sqrt{\lambda}}\int\left(-\lambda_0 + \frac{m(y)}{\alpha_1}\right)e^{\sqrt{\lambda}y}dy$$

(3.2.17)

Now B(y) =C. I+P.I (add 3.2.15 + 3.2.16)

$$B(y) = C_3 e^{\sqrt{\lambda}y} + C_4 e^{-\sqrt{\lambda}y} + \frac{e^{\sqrt{\lambda}y}}{2\sqrt{\lambda}}\int\left(-\lambda_0 + \frac{M(y)}{\alpha_1}\right)e^{-\sqrt{\lambda}y}dy - \frac{e^{-\sqrt{\lambda}y}}{2\sqrt{\lambda}}\int\left(-\lambda_0 + \frac{M(y)}{\alpha_1}\right)e^{\sqrt{\lambda}y}dy \qquad (3.2.18)$$

where $C_1, C_2, C_3 \,\&\, C_4$ are arbitrary constants.

Equation (3.1.11) utilizing equation (3.2.13) and applying method of separation of variables provides. Now considering equation (3.1.11) and substitute in (3.2.13) to obtain

$$\omega = -\nabla^2\psi$$

$$A(x) + B(y) = -\left[\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2}\right]$$

(3.2.19)

Add and subtract λ_1 in (3.2.19)

$$A(x) + \lambda_1 - \lambda_1 + B(y) = -\frac{d^2\psi}{dx^2} - \frac{d^2\psi}{dy^2}$$

(3.2.20)

Now separate variables of x and y in (3.2.20)

$$A(x) + \lambda_1 = -\frac{d^2\psi}{dx^2} \qquad (3.2.21)$$

$$-\lambda_1 + B(y) = -\frac{d^2\psi}{dy^2} \qquad (3.2.22)$$

Considering the variable of x in equations (3.2.21)

$$-\frac{d^2\psi}{dx^2} = A(x) - \lambda_1$$

$$\frac{d^2\psi}{dx} = (-A(x) + \lambda_1)dx$$

Integrate with respect to x from the above equations

$$\frac{d\psi}{dx} = -\int A(x)dx + \lambda_1 x + \lambda_2 \qquad (3.2.23)$$

Again ingrate with respect to x the equation (3.2.23)

$$\psi = -\int\int A(x)dxdx + \frac{\lambda_1 x^2}{2} + \lambda_2 x + \lambda_3 \qquad (3.2.24)$$

Now considering the variable of y in equation (3.2.22)

$$y_p = \frac{e^{\sqrt{\lambda}y}}{2\sqrt{\lambda}}\int\left(-\lambda_0 + \frac{m(y)}{\alpha_1}\right)e^{-\sqrt{\lambda}y}dy - \frac{e^{-\sqrt{\lambda}y}}{2\sqrt{\lambda}}\int\left(-\lambda_0 + \frac{m(y)}{\alpha_1}\right)e^{\sqrt{\lambda}y}dy$$

(3.2.17)

Now B(y) =C.I+P.I (add 3.2.15 + 3.2.16)

$$B(y) = C_3 e^{\sqrt{\lambda}y} + C_4 e^{-\sqrt{\lambda}y} + \frac{e^{\sqrt{\lambda}y}}{2\sqrt{\lambda}}\int\left(-\lambda_0 + \frac{M(y)}{\alpha_1}\right)e^{-\sqrt{\lambda}y}dy - \frac{e^{-\sqrt{\lambda}y}}{2\sqrt{\lambda}}\int\left(-\lambda_0 + \frac{M(y)}{\alpha_1}\right)e^{\sqrt{\lambda}y}dy \qquad (3.2.18)$$

where C_1, C_2, C_3 & C_4 are arbitrary constants.

Equation (3.1.11) utilizing equation (3.2.13) and applying method of separation of variables provides. Now considering equation (3.1.11) and substitute in (3.2.13) to obtain

$$\omega = -\nabla^2\psi$$

$$A(x) + B(y) = -\left[\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2}\right]$$

(3.2.19)

Add and subtract λ_1 in (3.2.19)

$$A(x) + \lambda_1 - \lambda_1 + B(y) = -\frac{d^2\psi}{dx^2} - \frac{d^2\psi}{dy^2}$$

(3.2.20)

Now separate variables of x and y in (3.2.20)

$$A(x) + \lambda_1 = -\frac{d^2\psi}{dx^2} \qquad (3.2.21)$$

$$-\lambda_1 + B(y) = -\frac{d^2\psi}{dy^2} \qquad (3.2.22)$$

Considering the variable of x in equations (3.2.21)

$$-\frac{d^2\psi}{dx^2} = A(x) - \lambda_1$$

$$\frac{d^2\psi}{dx} = (-A(x) + \lambda_1)dx$$

Integrate with respect to x from the above equations

$$\frac{d\psi}{dx} = -\int A(x)dx + \lambda_1 x + \lambda_2 \qquad (3.2.23)$$

Again ingrate with respect to x the equation (3.2.23)

$$\psi = -\int\int A(x)dxdx + \frac{\lambda_1 x^2}{2} + \lambda_2 x + \lambda_3 \quad (3.2.24)$$

Now considering the variable of y in equation (3.2.22)

$$-\frac{d^2\psi}{dy^2} = B(y) + \lambda_1$$

$$\frac{d^2\psi}{dy^2} = -B(y) - \lambda_1$$

$$\frac{d^2\psi}{dy} = (-B(y) - \lambda_1)dy$$

Integrate with respect to y the above equations

$$\frac{d\psi}{dy} = -\int B(y)dy - \lambda_1 y + \lambda_3 \quad (3.2.25)$$

Again Integrate with respect to y the above equation

$$\psi = -\int\int B(y)dydy - \frac{\lambda_1 y^2}{2} + \lambda_4 y + \lambda_5 \quad (3.2.26)$$

Now adding the equations (3.2.24) + (3.2.26)

$$\psi = \frac{\lambda_1}{2}(x^2 - y^2) + \lambda_2 x + \lambda_4 y - \iint A(x)dxdx - \iint B(y)dydy + \lambda_3 + \lambda_5 \quad (3.2.27)$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and λ_5 are arbitrary real constants.

2.2 General Solution of Force Components

In order to determine force components f_1 and f_2, And insert equations (3.2.1-3.2.3, 3.2.27) in equations (3.1.12) and (3.1.13) to obtain general solution of force components.
Now consider the equation (3.1.12)

$$h_x = -\rho\psi_x\omega - \mu\omega_y + \alpha_1\psi_x\nabla^2\omega + F_1$$

$$[\because \omega = A(x) + B(x)]$$

$$= -B'(y)\mu + (\alpha_1\psi_x\nabla^2\omega - \rho\psi_x\omega) + F_1$$

$$[\because \omega_y = B'(y), \omega_x = A'(x)] \qquad = -\mu B'(y) + \psi_x(\alpha_1\nabla^2\omega - \rho\omega) + F_1$$

$$[\because \alpha_1\nabla^2\omega - \rho\omega = G(x) + M(y)]$$

Substitute the value of ψ_x from (3.2.23) in the above equation obtain

$$h_x = -\mu B'(y) + \left(\lambda_1 x + \lambda_2 - \int A(x)dx\right)[G(x) + M(y)] + F_1 \quad (3.3.1)$$

$$h_x = -\mu B'(y) + F_1^* \quad (3.3.2)$$

where $$F_1^* = \left(\lambda_1 x + \lambda_2 - \int A(x)dx\right)[G(x) + M(y)] + F_1 \quad (3.3.3)$$

Now consider (3.1.13)

$$h_y = -\rho\psi_y\omega + \mu\omega_x + \alpha_1\psi_y\nabla^2\omega + F_2 \qquad [\because \omega = A(x) + B(x)]$$

$$= \mu A'(x) + \psi_y(\alpha_1\nabla^2\omega - \rho\omega) + F_2$$

$$[\because \omega_y = B'(y), \omega_x = A'(x)]$$

Substitute the value of ψ_y from (3.2.25) in the above equation to get the solution for y

$$h_y = \mu A'(x) + \left(\lambda_1 y + \lambda_4 - \int B(y)dy\right)\left[G(x) + M(y)\right] + F_2$$

(3.3.4)

$$[\because \alpha_1 \nabla^2 \omega - \rho\omega = G(x) + M(y)]$$

$$h_y = \mu A'(x) + F_{2.}^* \quad (3.3.5)$$

where $F_2^* = \left(\lambda_1 y + \lambda_4 - \int B(y)dy\right)\left[G(x) + M(y)\right] + F_2$

(3.3.6)

Now consider equation (3.3.2)

$$h_x = -\mu B'(y) + F_1^*$$

differentiate with respect to y the above equation

$$h_{xy} = -\mu B''(y) + F_1^*{}_y \quad (3.3.7)$$

Now consider equation (3.3.5) is

$$h_y = \mu A'(x) + F_{2.}^*.$$

differentiate with respect to x the above equation

$$h_{yx} = \mu A''(x) + F_{2.}^*{}_x \quad (3.3.8)$$

Applying inerrability condition $h_{xy} = h_{yx}$ on equations (3.3.7) and (3.3.8) to get

$$-\mu B''(y) + F_1^*{}_y = \mu A''(x) + F_{2.}^*{}_x \quad (3.3.9)$$

Now substitute $F_1^* = D_1(x) + D_2(y)$ (3.3.10)

Differentiate with respect to y the above equation

$$F_1^*{}_y = D_2'(y) \quad (3.3.11)$$

and again substitute

$$F_2^* = D_3(x) + D_4(y) \quad (3.3.12)$$

Differentiate equation (3.3.12) with respect to x $F_2^*{}_x = D_3'(x)$ (3.3.13)

Put $F_1^*{}_y$ and $F_1^*{}_x$ values in (3.3.9)

$$-\mu B''(y) + D_2'(y) = \mu A''(x) + D_3'(x) = d_1$$

(3.3.14)

Now separate the variable y in (3.5.14)

$$-\mu B''(y) + D_2'(y) = d_1$$

$$D_2'(y) = \mu B''(y) + d_1 \quad (3.3.15)$$

Integrate equation (3.3.15) with respect to y

$$D_2(y) = \mu B'(y) + d_1 y + d_2 \quad (3.3.16)$$

Now consider x variable in (3.3.14)

$$\mu A''(x) + D_3'(x) = d_1 \quad (3.3.17)$$

Integrate equation (3.3.17) with respect to x

$$D_3(x) = -\mu A'(x) + d_1 x + d_3 \quad (3.3.18)$$

d_1, d_2 and d_3 are arbitrary real constant.

Now considering the force components equation (3.1.14)

$$F_1 = \rho f_1$$

Consider (3.3.3) to obtain the value of F_1

$$\rho f_1 = F_1$$

$= F_1^* - \left(\lambda_1 x + \lambda_2 - \int A(x)dx\right)(G(x) + M(y))$ Substitute equation (3.3.10) and (3.3.16) in above equation

$$\rho f_1 = -\left(\lambda_1 x + \lambda_2 - \int A(x)dx\right) z(x,y) + D_1(x) + \mu B'(y) + d_1 y + d_2$$

(3.3.19)

Now considering the force components equation (3.1.14)

$$F_2 = \rho f_2$$

Consider (3.3.6) to get the value of F2

$$\rho f_2 = F_2 = F_2^* - \left(\lambda_1 y + \lambda_4 - \int B(y)dy\right)$$
$$(G(x) + M(y))$$

Substitute equation (3.3.12) and (3.3.18) in above equation

$$\rho f_2 = -\left(-\lambda_1 y + \lambda_4 - \int B(y)dy\right) z(x, y) +$$
$$D_4(y) - \mu A'(x) + d_1 x + d_3$$
(3.3.20)

The equations (3.3.19) and (3.3.20) are general solutions of the force components where d_1, d_2, d_3, d_4 are real constants.

2.3 General Solution of Steady Plane Flow

Consider equation (3.3.2)

$$h_x = -\mu B'(y) + F_1^*$$

Substitute equation (3.3.10) in the above equation

$$h_x = -\mu B'(y) + D_1(x) + D_2(y) \qquad (3.4.1)$$

Substitute the equation (3.3.16) in the above equation

$$h_x = -\mu B'(y) + D_1(x) + \mu B'(y) + d_1 y + d_2$$
$$h_x = D_1(x) + d_1 y + d_2 \qquad (3.4.2)$$

Integrate equation (3.4.2) with respect to x

$$h = \int D_1(x)dx + d_1 yx + d_2 x \qquad (3.4.3)$$

Now consider equation (3.3.5)

$$h_y = \mu A'(y) + F_2^*$$

Substitute equation (3.3.12) in the above equation and get

$$h_y = \mu A'(y) + D_3(x) + D_4(y) \qquad (3.4.4)$$

Substitute the equation (3.3.18) in the above equation

$$h_y = -\mu A'(x) + D_4(x) + \mu A'(y) + d_1 x + d_3$$
$$h_y = D_4(y) + d_1 x + d_3 \qquad (3.4.5)$$

Integrate equation (3.4.5) with respect to y

$$h = \int D_4(y)dy + d_1 xy + d_3 y \qquad (3.4.6)$$

Now adding the equations (3.4.3) + (3.4.6)

$$h = \int D_1(x)dx + \int D_4(y)dy + d_1 xy + d_2 x + d_3 y + d_4 \qquad (3.4.7)$$

where d_4 is an arbitrary real constant, and $D_1(x)$, $D_4(y)$ are arbitrary functions. The pressure p can easily be determined through equation (3.1.5) employing equation (3.4.7).

The general solution to equations (3.1.6-3.1.9) is

$$\psi = \frac{\lambda_1}{2}(x^2 - y^2) + \lambda_2 x + \lambda_4 y - \iint A(x)dxdx -$$
$$\iint B(y)dydy + \lambda_3 + \lambda_5 \qquad (3.4.8)$$

$$u = -\lambda_1 y + \lambda_4 - \int B(y)dy \qquad (3.4.9)$$

$$v = -\lambda_1 x + \lambda_2 - \int A(x)dx \qquad (3.4.10)$$

$$\rho f_1 = -\left(\lambda_1 x + \lambda_2 - \int A(x)dx\right)z(x,y) + D_1(x) + \mu B'(y) + d_1 y + d_2 \quad (3.4.11)$$

$$\rho f_2 = -\left(-\lambda_1 y + \lambda_4 - \int B(y)dy\right)z(x,y) + D_4(y) - \mu A'(x) + d_1 x + d_3 \quad (3.4.12)$$

$$h = \int D_1(x)dx + \int D_4(y)dy + d_1 xy + d_2 x + d_3 y + d_4$$

(3.4.13) where A(x) and B(y) are given by equations (3.2.7) and (3.2.8), respectively.

2.4 Special Cases of the General Solutions of Stream Functions

The general solution of equations describing the steady plane flows of incompressible second grade fluids in the presence of unknown body force is determined when vorticity function ω satisfies the partial differential equation in equation (3.2.1). The general solution contains four arbitrary functions G(x), M(y), $D_1(x)$ and $D_4(y)$. The arbitrariness of these four functions indicates that an infinite set of exact solutions can be generated for flow equations (3.1.6-3.1.9). and constructed some stream functions ψ for some forms of the functions G(x) and M(y) These stream functions ψ are

1. For G(x)=n, M(y)=m

$$\psi = \beta_1 x^2 - \beta_2 y^2 + \lambda_2 x + \lambda_4 y - \frac{1}{\lambda}\left(C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x}\right) - \frac{1}{\lambda}\left(C_3 e^{\sqrt{\lambda}y} + C_4 e^{-\sqrt{\lambda}y}\right) + \beta_3 \quad (3.5.1)$$

2. For G(x)= nx, M(y)=my

$$\psi = \beta_4 x^2 - \beta_5 y^2 + \lambda_2 x + \lambda_4 y - \frac{1}{\lambda}\left(C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x}\right) + \beta_6 x^3 - \beta_7 y^3 - \frac{1}{\lambda}\left(C_3 e^{\sqrt{\lambda}y} + C_4 e^{-\sqrt{\lambda}y}\right) + \beta_3 \quad (3.5.2)$$

3. For $G(x) = n_1 \cos(n_2 x + n_3)$, $M(y) = m_1 \cos(m_2 y + m_3)$

$$\psi = \beta_8 x^2 - \beta_8 y^2 + \lambda_2 x + \lambda_4 y - \frac{1}{\lambda}\left(C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x}\right) - \beta_9 \cos(n_2 x + n_3) - \beta_{10}\cos(m_2 x + m_3) - \frac{1}{\lambda}\left(C_3 e^{\sqrt{\lambda}y} + C_4 e^{-\sqrt{\lambda}y}\right) + \beta_3 \quad (3.5.3)$$

where the parameter $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8, \beta_9, \beta_{10}$ are given in appendix.

2.5 Stream Lines in the Porous Medium

Now let us consider the constant parameter having the porous characteristics to design the stream lines in the porous medium.

1.Considering the equation (3.5.1, 3.5.2 & 3.5.3) in this substitute the values of the parameter for $\lambda_2 = \frac{u}{k'}$, $\lambda_4 = \frac{v}{k'}$.

$$\psi = \beta_1 x^2 - \beta_2 y^2 + \frac{u}{k'}x + \frac{v}{k'}y - \frac{1}{\lambda}\left(C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x}\right) - \frac{1}{\lambda}\left(C_3 e^{\sqrt{\lambda}y} + C_4 e^{-\sqrt{\lambda}y}\right) + \beta_3 \quad (3.6.1)$$

$$\psi = \beta_4 x^2 - \beta_5 y^2 + \frac{u}{k'}x + \frac{v}{k'}y - \frac{1}{\lambda}\left(C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x}\right) + \beta_6 x^3 - \beta_7 y^3 - \frac{1}{\lambda}\left(C_3 e^{\sqrt{\lambda}y} + C_4 e^{-\sqrt{\lambda}y}\right) + \beta_3 \quad (3.6.2)$$

$$\psi = \beta_8 x^2 - \beta_8 y^2 + \frac{u}{k'}x + \frac{v}{k'}y - \frac{1}{\lambda}\left(C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x}\right) - \beta_9 \cos(n_2 x + n_3) - \beta_{10}\cos(m_2 x + m_3) - \frac{1}{\lambda}\left(C_3 e^{\sqrt{\lambda}y} + C_4 e^{-\sqrt{\lambda}y}\right) + \beta_3 \quad (3.6.3)$$

Where permeability coefficient is k', u, v are velocity components along the x direction and y direction.

2. Now substitute the values of the parameter $\lambda_2 = \frac{1}{k'}$, $\lambda_4 = \frac{1}{k'}$ in (3.5.1, 3.5.2 & 3.5.3) if permeability condition is increase and neglecting the velocity u, v.

$$\psi = \beta_1 x^2 - \beta_2 y^2 + \frac{1}{k'}x + \frac{1}{k'}y - \frac{1}{\lambda}\left(C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x}\right) - \frac{1}{\lambda}\left(C_3 e^{\sqrt{\lambda}y} + C_4 e^{-\sqrt{\lambda}y}\right) + \beta_3 \tag{3.6.4}$$

$$\psi = \beta_4 x^2 - \beta_5 y^2 + \frac{1}{k'}x + \frac{1}{k'}y - \frac{1}{\lambda}\left(C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x}\right) + \beta_6 x^3 - \beta_7 y^3 - \frac{1}{\lambda}\left(C_3 e^{\sqrt{\lambda}y} + C_4 e^{-\sqrt{\lambda}y}\right) + \beta_3 \tag{3.6.5}$$

$$\psi = \beta_8 x^2 - \beta_8 y^2 + \frac{1}{k'}x + \frac{1}{k'}y - \frac{1}{\lambda}\left(C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x}\right) - \beta_9 \cos(n_2 x + n_3) - \beta_{10}\cos(m_2 x + m_3) - \frac{1}{\lambda}\left(C_3 e^{\sqrt{\lambda}y} + C_4 e^{-\sqrt{\lambda}y}\right) + \beta_3 \tag{3.6.6}$$

3.Now consider the parameter $\lambda_2 = \frac{hK_F}{\varepsilon K_M}$ and $\lambda_4 = \frac{hK_F}{\varepsilon K_M}$ where K_F is fracture permeability and K_M is matrix permeability substitute in (3.5.1, 3.5.2 & 3.5.3)

$$\psi = \beta_1 x^2 - \beta_2 y^2 + \frac{hK_F}{\varepsilon K_M}x + \frac{hK_F}{\varepsilon K_M}y - \frac{1}{\lambda}\left(C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x}\right) - \frac{1}{\lambda}\left(C_3 e^{\sqrt{\lambda}y} + C_4 e^{-\sqrt{\lambda}y}\right) + \beta_3 \tag{3.6.7}$$

$$\psi = \beta_4 x^2 - \beta_5 y^2 + \frac{hK_F}{\varepsilon K_M}x + \frac{hK_F}{\varepsilon K_M}y - \frac{1}{\lambda}\left(C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x}\right) + \beta_6 x^3 - \beta_7 y^3 - \frac{1}{\lambda}\left(C_3 e^{\sqrt{\lambda}y} + C_4 e^{-\sqrt{\lambda}y}\right) + \beta_3 \tag{3.6.8}$$

$$\psi = \beta_8 x^2 - \beta_8 y^2 + \frac{hK_F}{\varepsilon K_M}x + \frac{hK_F}{\varepsilon K_M}y - \frac{1}{\lambda}\left(C_1 e^{\sqrt{\lambda}x} + C_2 e^{-\sqrt{\lambda}x}\right) - \beta_9 \cos(n_2 x + n_3) - \beta_{10}\cos(m_2 x + m_3) - \frac{1}{\lambda}\left(C_3 e^{\sqrt{\lambda}y} + C_4 e^{-\sqrt{\lambda}y}\right) + \beta_3 \tag{3.6.9}$$

3 RESULTS AND DISCUSSION

The stream lines for this stream function ψ are presented through figures (1-32).

Figure (1-10) depict stream lines for the stream function ψ given by equation (3.5.1) for various values of the parameters therein. It is observe that streamlines are closed curves as long as the quadratic terms dominate other terms. Figures (1-3) show the effect on the streamlines due to the change in the value of parameter β_3 for fixed values of other parameters. Figures (1) and (4) show the effect on the streamlines due to change in the values of the parameters β_1 and β_2 for fixed values of the other parameters. Figure (2) and (3) illustrate the influence of the parameter λ_4 on the streamlines patterns. Figures (6) and (7) indicates the effect on streamlines due to change in the value of parameter β_2 figures (8-10) depict the nature of streamlines for various values of the parameters.

Figures (11-16) represent streamlines for the stream function ψ in equation (3.5.2) for various values of the parameters in equation (3.5.2). The effect of various values of the parameters on streamlines is obvious for the figures.

Figures (17-23) represent the nature of stream lines for the stream function ψ in equation (3.5.3). Figure (17) and (18) depict change in the stream lines due to change in the value of for fixed values of ψ the other parameters. Figures (18) and (19) indicate the influence of the parameter C_3. Figures (20-23) show the effect of the parameter β_{10} on the stream lines for fixed value of other parameters. Figure (23) indicates the effect of the parameters C_3, C_4, β_9 and β_{10} on the streamlines in figure (22) for fixed values of the other parameters.

Figure (24-26) represent the streamlines for the equations (3.6.1 – 3.6.3) where the parameter λ_2 and λ_4 having the characteristics of permeability. And the value of permeability coefficient is $k' \leq 0.6$.

Figure (27-29) represent the stream lines of the equations (3.6.4 – 3.6..6) when neglecting the fluid velocity. And the value of permeability coefficient is $k' \leq 0.6$,

Figure (30-32) indicates the stream lines of the equations (3.6.7 – 3.6.9). When the parameter $\lambda_2 = \frac{hK_F}{\varepsilon K_M}$ and $\lambda_4 = \frac{hK_F}{\varepsilon K_M}$ where K_F is fracture permeability and K_M is matrix permeability. And the values of permeability $K_F = 10^3, K_M = 1, \varepsilon = 10^{-4}$ and $h = 10$ due to the fracture permeability figure (30-32) does not having stream lines.

3.1 STREAMLINES PATTERN FOR STEADY PLANE FLOW

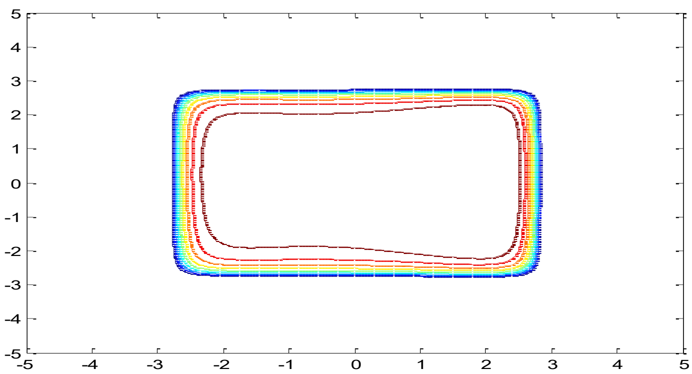

Figure **1**: β_1=5.186, β_2=3.186, β_3=128, λ_2=6, λ_4=2, λ=4, $C_1=C_2=C_3=C_4$= 1

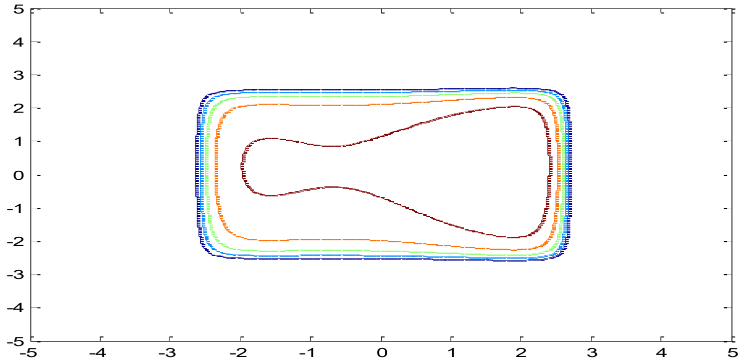

Figure **2:** β_1=5.186, β_2=3.186, β_3=6, λ_2=6, λ_4=2, λ=4, $C_1=C_2=C_3=C_4$= 1

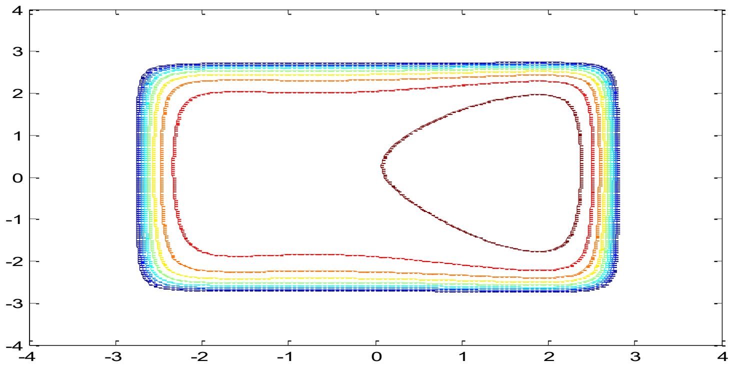

Figure **3**: β_1=5.186, β_2=3.186, β_3=2, λ_2=6, λ_4=2, λ=4, C_1=C_2=C_3=C_4= 1

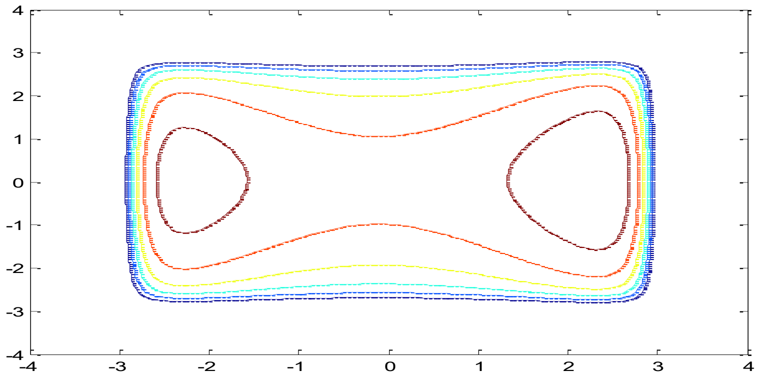

Figure 4: β_1=25.186, β_2=23.186, β_3=128, λ_2=6, λ_4=2, λ=4, C_1= C_2=C_3= C_4= 1

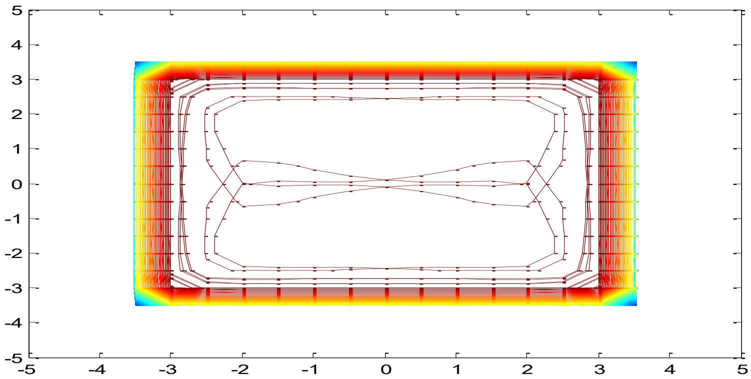

Figure 5: β_1=5.186, β_2=3.186, β_3=6, λ_2=6, λ_4=32, λ=4, C_1= C_2= C_3= C_4= 1

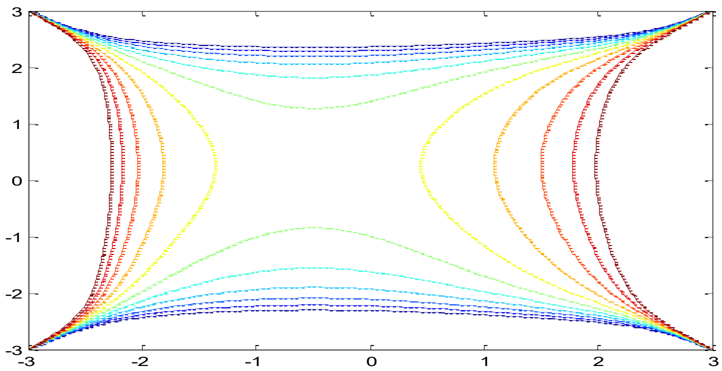

Figure 6: β_1=5.186, β_2=3.186, β_3=28, λ_2=6, λ_4=2, λ=4,C_1= C_2= 1, C_3= C_4= -1

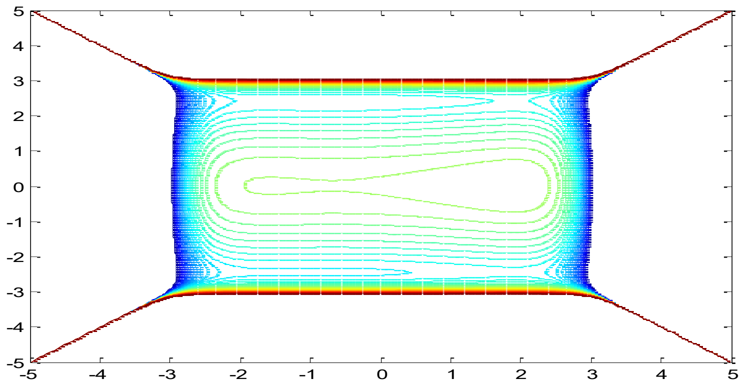

Figure 7: β_1=5.186, β_2=43.186, β_3=28, λ_2=6, λ_4=2, λ=4,C_1= C_2=1, C_3= C_4= -1

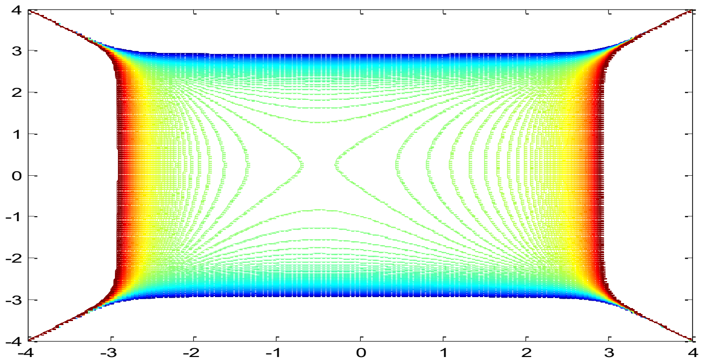

Figure 8: β_1=5.186, β_2=3.186, β_3=6, λ_2=6, λ_4=2, λ=4,C_1= C_2= -1, C_3= C_4= 1

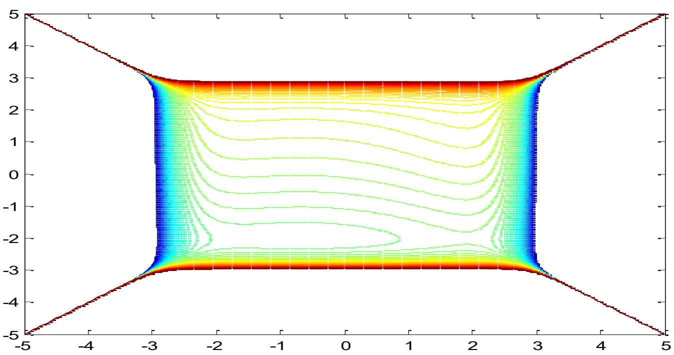

Figure 9: β_1=5.186, β_2=3.186, β_3=28, λ_2=6, λ_4=28, λ=4, C_1=C_2=1, C_3=C_4= -1

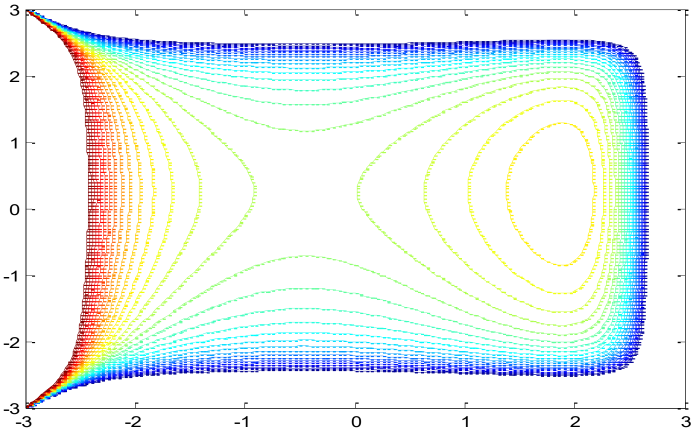

Figure 10: β_1=5.186, β_2=3.186, β_3=6, λ_2=6, λ_4=2, λ=4,C_1=1, C_2=-1, C_3=C_4=1

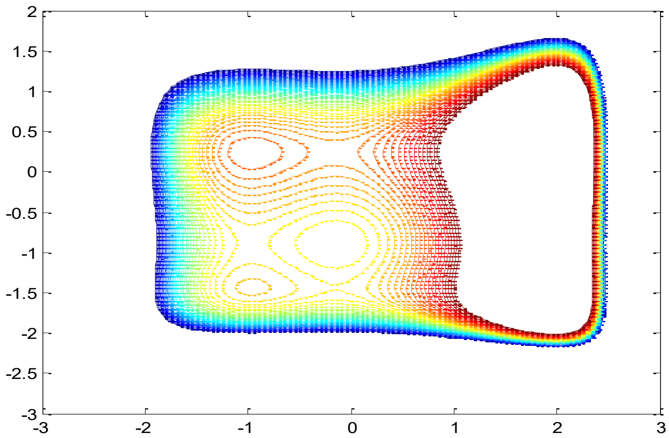

Figure **11:** Streamlines pattern for β_3 = 15, β_4 = 35,

β_5 = 12, β_6 = 12, β_7 = 35, λ = 4, λ_2= 6, λ_4=15, C_1= 8,
C_2= 12, C_3= 16, C_4 =24

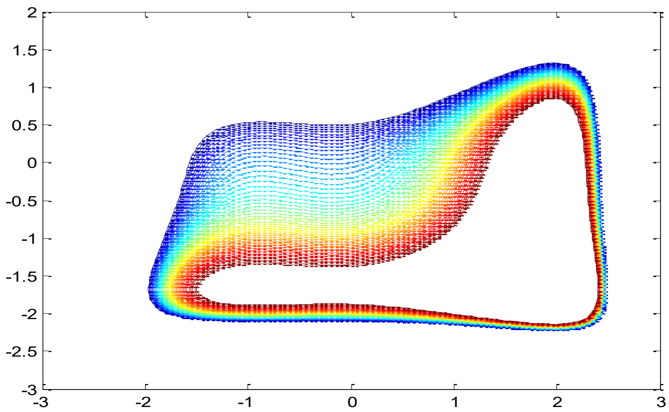

Figure **12:** Streamlines pattern for β_3 = 15, β_4 = 32,

β_5 = 2, β_6 = 12, β_7 = 35, λ = 4, λ_2 = 6,
λ_4 = -35, C_1 =8, C_2= 12, C_3= 16, C_4 =24

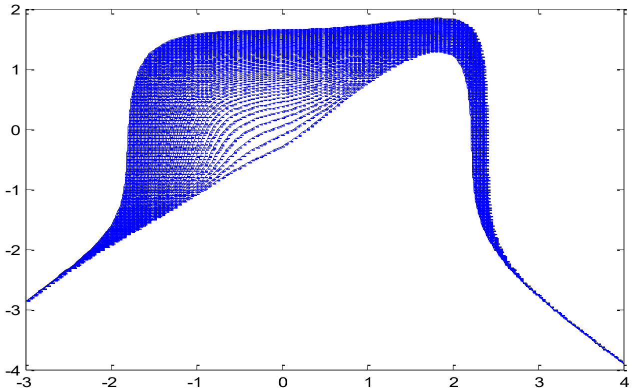

Figure **13:** Streamlines pattern for β_3= 15, β_4= 12,

β_5 = 12, β_6 = 12, β_7 = 3, λ = 4, λ_2 = 6, λ_4 = 6,
C_1 = 8, C_2 =12, C_3= 16, C_4 = -24

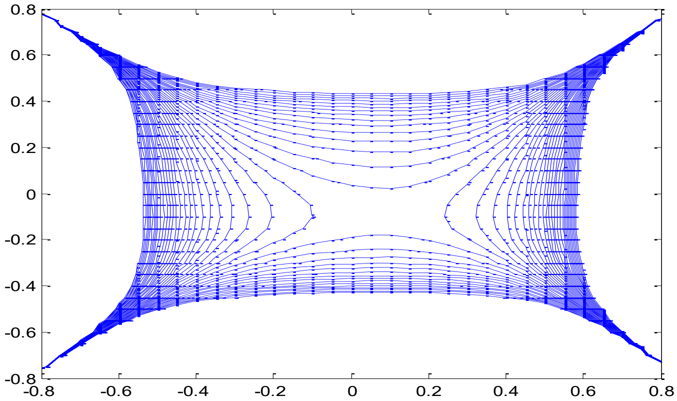

Figure **14:** Streamlines pattern for β_3 = 15, β_4 = 5,

β_5 = 12, β_6= 12, β_7= 43, λ = 144, λ_2 = 6,
λ_4 = 46, C_1 = 2 λ, C_2 = 3 λ, C_3= - 4 λ , C_4 = - 6 λ.

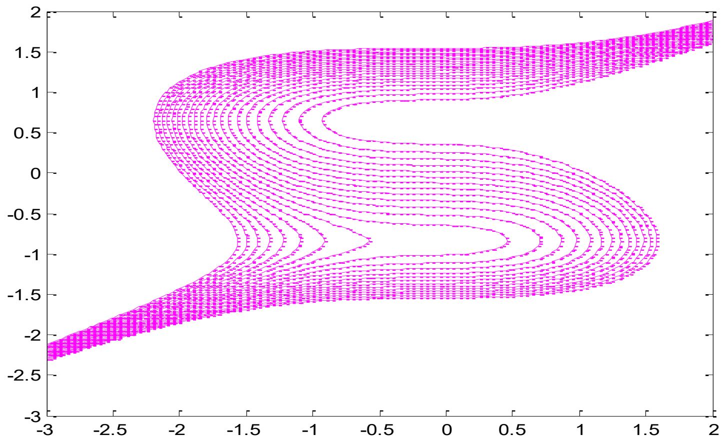

Figure **15:** Streamlines pattern for β_3 = 15, β_4 = 5,

β_5 = 12, β_6 = 12, β_7= 43, λ = 1, λ_2 = 2, λ_4 = 70, C_1 = 2,
C_2 = 3, C_3= - 4, C_4= - 6.

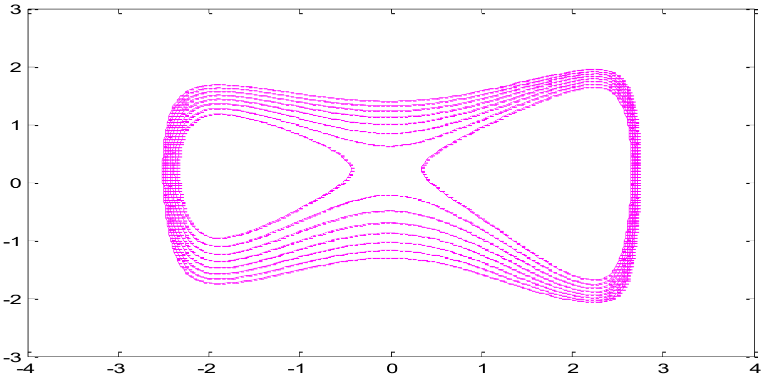

Figure **16:** Streamlines pattern for $\beta_3 = 15$, $\beta_4 = 120$,

$\beta_5 = 150$, $\beta_6 = 12$, $\beta_7 = 35$, $\lambda = 4$, $\lambda_2 = 6$,
$\lambda_4 = 75$, $C_1 = 8$, $C_2 = 12$, $C_3 = 16$, $C_4 = 24$.

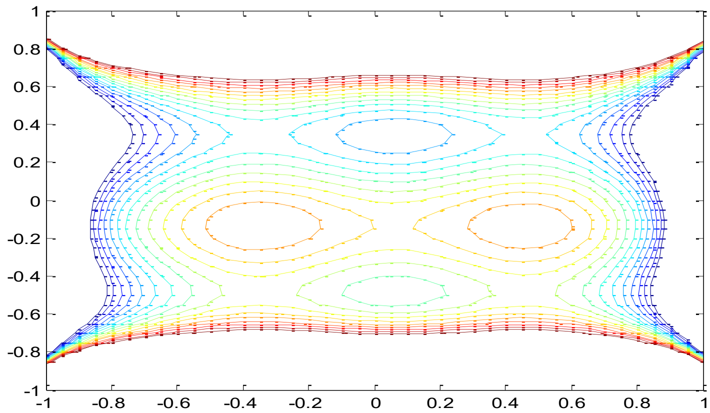

Figure **17:** Streamlines pattern for $\beta_3 = 15$, $\beta_8 = 10$,

$\beta_9 = -120$, $\beta_{10} = -400$, $\lambda = 36$, $\lambda_2 = 15$,
$\lambda_4 = 19$, $C_1 = 8\lambda$, $C_2 = 9\lambda$, $C_3 = -42\lambda$, $C_4 = -43\lambda$, $n_2 = 7$, $n_3 = 9$, $m_2 = 6$, $m_3 = 7$

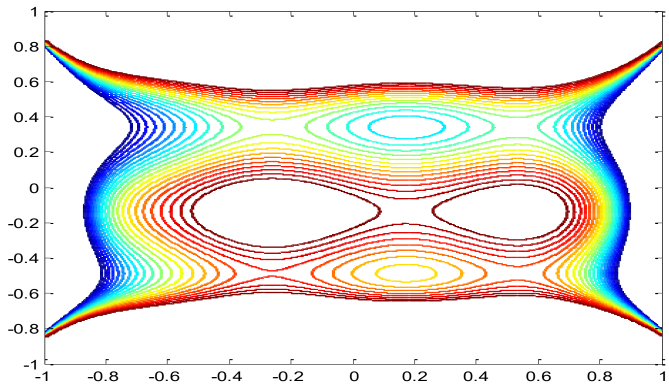

Figure **18:** Streamlines pattern for $\beta_3 = 15$, $\beta_8 = 10$,

$\beta_9 = -120$, $\beta_{10} = -400$, $\lambda = 36$, $\lambda_2 = 15$,
$\lambda_4 = 19$, $C_1 = 8\lambda$, $C_2 = 9\lambda$, $C_3 = -42\lambda$, $C_4 = -43\lambda$, $n_2 = 7$, $n_3 = 2$, $m_2 = 6$, $m_3 = 7$.

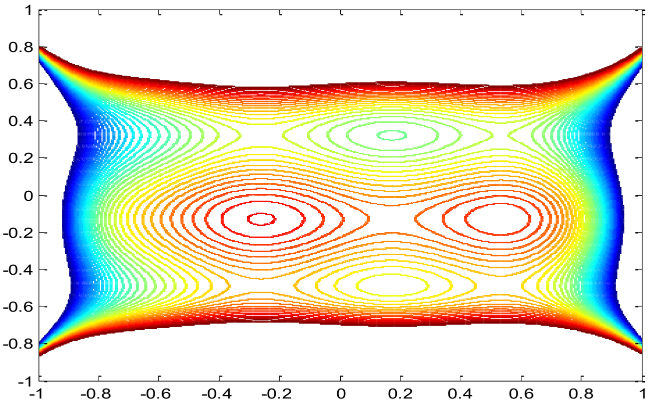

Figure 19: Streamlines pattern for $\beta_3 = 15$, $\beta_8 = 10$,

$\beta_9 = -120$, $\beta_{10} = -400$, $\lambda = 36$, $\lambda_2 = 15$,
$\lambda_4 = 19$, $C_1 = 8\lambda$, $C_2 = 9\lambda$, $C_3 = -62\lambda$, $C_4 = -43\lambda$, $n_2 = 7$, $n_3 = 2$, $m_2 = 6$, $m_3 = 7$.

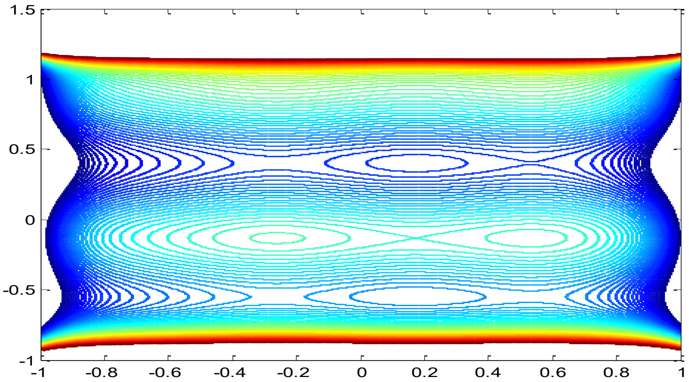

Figure **20:** Streamlines pattern for $\beta_3 = 15$, $\beta_8 = 10$,

$\beta_9 = -120$, $\beta_{10} = -900$, $\lambda = 36$, $\lambda_2 = 15$,
$\lambda_4 = 19$, $C_1 = 8\lambda$, $C_2 = 9\lambda$, $C_3 = -2\lambda$, $C_4 = -43\lambda$, $n_2 = 7$, $n_3 = 2$, $m_2 = 6$, $m_3 = 7$.

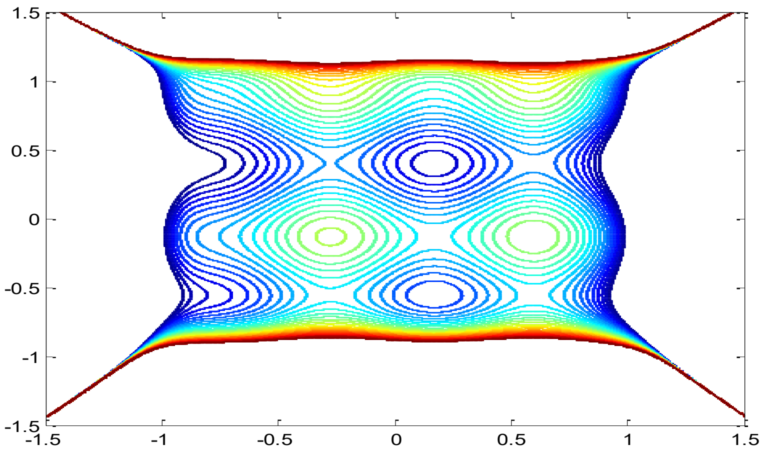

Figure **21:** Streamlines pattern for $\beta_3 = 15$, $\beta_8 = 10$,

$\beta_9 = -720$, $\beta_{10} = -1000$, $\lambda = 36$, $\lambda_2 = 15$, $\lambda_4 = 19$, $C_1 = 8\lambda$, $C_2 = 9\lambda$, $C_3 = -2\lambda$, $C_4 = -43\lambda$, $n_2 = 7$, $n_3 = 2$, $m_2 = 6$, $m_3 = 7$.

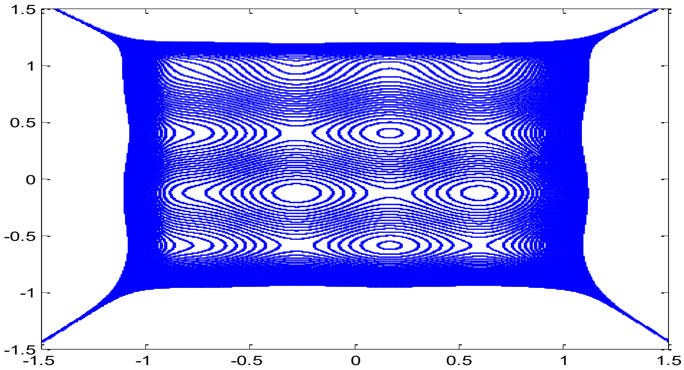

Figure **22:** Streamlines pattern for $\beta_3 = 15$, $\beta_8 = 10$,

$\beta_9 = -720$, $\beta_{10} = -1900$, $\lambda = 36$, $\lambda_2 = 15$, $\lambda_4 = 19$, $C_1 = 8\lambda$, $C_2 = 9\lambda$, $C_3 = -2\lambda$, $C_4 = -43\lambda$, $n_2 = 7$, $n_3 = 2$, $m_2 = 6$, $m_3 = 7$.

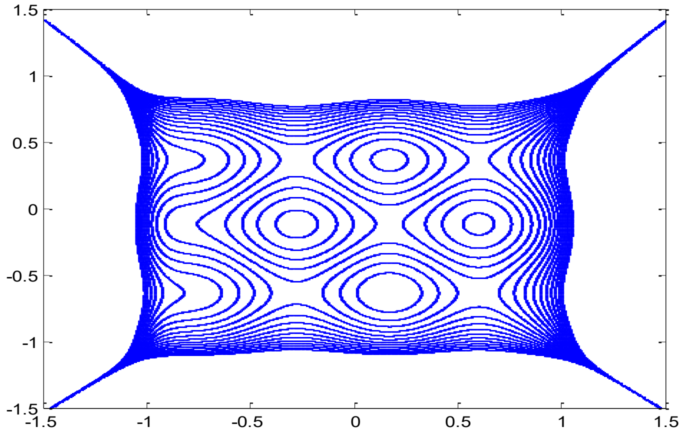

Figure **23:** Streamlines pattern for $\beta_3 = 15$, $\beta_8 = 10$,

$\beta_9 = -920$, $\beta_{10} = -1200$, $\lambda = 36$, $\lambda_2 = 15$ $\lambda_4 = 19$, $C_1 = 8\lambda$, $C_2 = 9\lambda$, $C_3 = -64\lambda$, $C_4 = -4\lambda$, $n_2 = 7$, $n_3 = 2$, $m_2 = 6$, $m_3 = 7$.

STREAMLINES PATTERN FOR POROUS PLANE

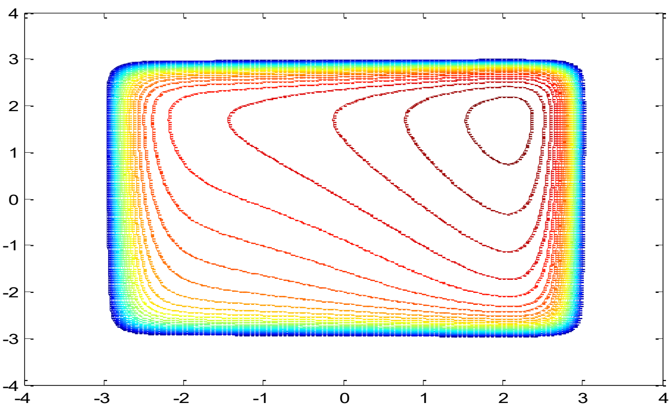

Figure **24:** $\beta_1=5.186$, $\beta_2=3.186$, $\beta_3=128$, $k'=0.6$, $u=5$, $v=5$, $\lambda=4$, $C_1=C_2=C_3=C_4=1$

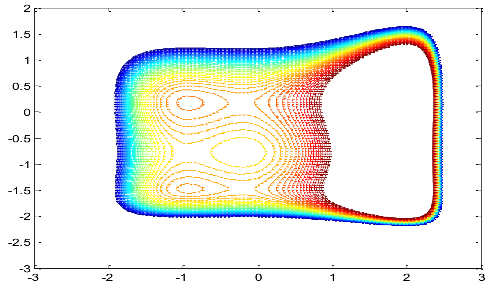

Figure 25: Streamlines pattern for $\beta_3 = 15$, $\beta_4 = 35$,
$\beta_5 = 12$, $\beta_6 = 12$, $\beta_7 = 35$, $\lambda = 4$, k'=0.6, u=v=5, $C_1 = 8$, $C_2 = 12$, $C_3 = 16$, $C_4 = 24$

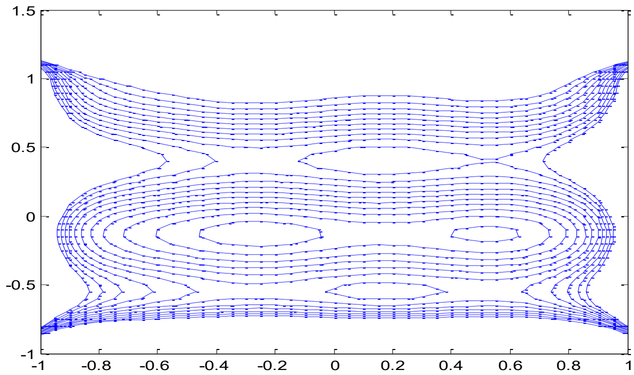

Figure **26:** Streamlines pattern for $\beta_3 = 15$, $\beta_8 = 10$,

$\beta_9 = -120$, $\beta_{10} = -900$, $\lambda = 36$, u=v=5, k'=0.6, $C_1 = 8\lambda$, $C_2 = 9\lambda$, $C_3 = -2\lambda$, $C_4 = -43\lambda$, $n_2 = 7$, $n_3 = 2$, $m_2 = 6$, $m_3 = 7$.

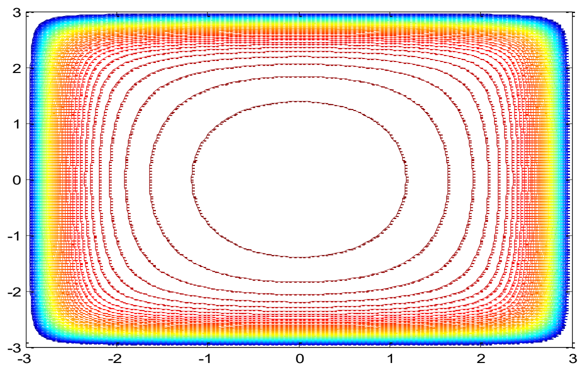

Figure **27:** $\beta_1 = -5.186$, $\beta_2 = 3.186$, $\beta_3 = 128$, k'=0.6, $\lambda=4$, $C_1=C_2=C_3=C_4=1$

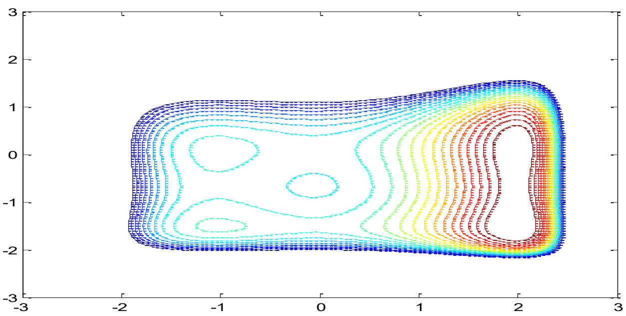

Figure 28: Streamlines pattern for $\beta_3 = 15$, $\beta_4 = 35$,

$\beta_5 = 12$, $\beta_6 = 12$, $\beta_7 = 35$, $\lambda = 4$, k'=0.6,
$C_1 = 8$, $C_2 = 12$, $C_3 = 16$, $C_4 = 24$

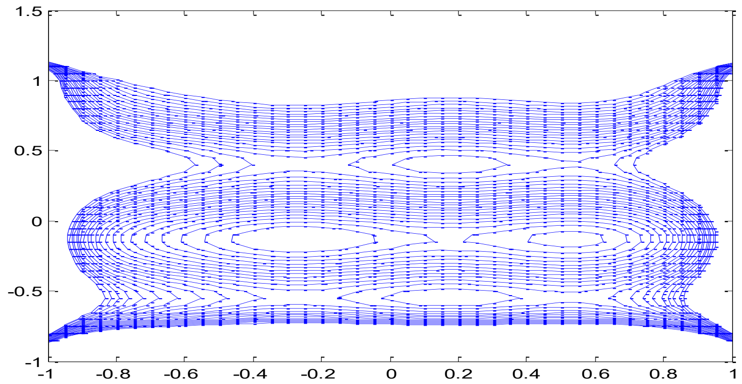

Figure 29: Streamlines pattern for $\beta_3 = 15$, $\beta_8 = 10$,

$\beta_9 = -120$, $\beta_{10} = -900$, $\lambda = 36$, $k' = 0.6$,

- ✓ $C_1 = 8\lambda$, $C_2 = 9\lambda$, $C_3 = -2\lambda$, $C_4 = -43\lambda$, $n_2 = 7$, Some stream line patterns are studied for the second-grade fluid in the presence of body force, neglecting the thermal effects.

$n_3 = 2$, $m_2 = 6$, $m_3 = 7$.

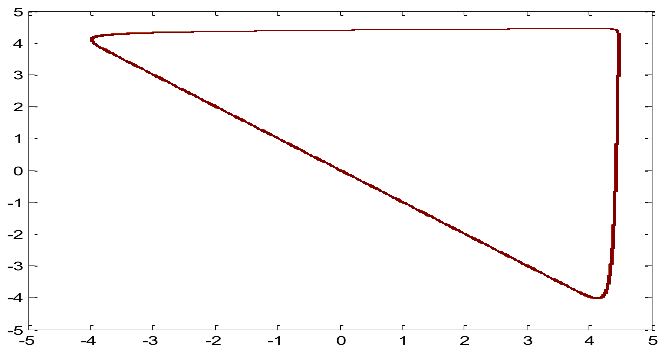

Figure 30: $\beta_1 = 5.186$, $\beta_2 = 3.186$, $\beta_3 = 128$, $h = 10$, $k_F = 10^3$, $k_M = 1$, $\varepsilon = 10^{-4}$, $\lambda = 4$, $C_1 = C_2 = C_3 = C_4 = 1$

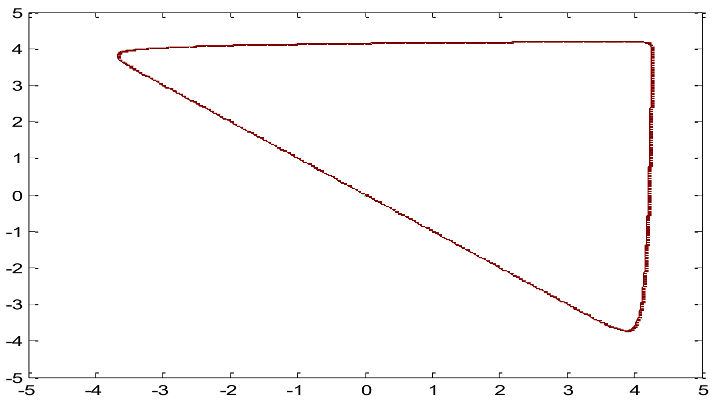

Figure 31: Streamlines pattern for $\beta_3 = 15$, $\beta_4 = 35$,

$\beta_5 = 12$, $\beta_6 = 12$, $\beta_7 = 35$, $\lambda = 4$, $h = 10$,
$k_F = 10^3$, $k_M = 1$, $\varepsilon = 10^{-4}$, $C_1 = 8$, $C_2 = 12$, $C_3 = 16$,
$C_4 = 24$

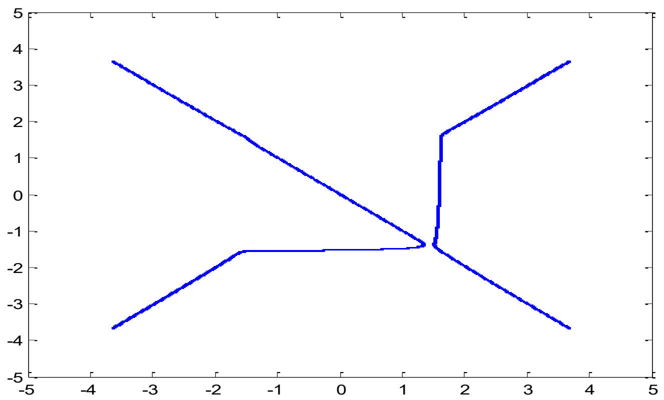

Figure 32: Streamlines pattern for $\beta_3 = 15$, $\beta_8 = 10$,

$\beta_9 = -120$, $\beta_{10} = -900$, $\lambda = 36$, h=10,
$k_F = 10^3$, $k_M = 1$, $\varepsilon = 10^{-4}$, $C_1 = 8\lambda$, $C_2 = 9\lambda$, $C_3 = -2\lambda$, $C_4 = -43\lambda$, $n_2 = 7$, $n_3 = 2$, $m_2 = 6$, $m_3 = 7$.

4 ONCLUSION

- ✓ The effect of various second grade parameters on flow pattern is studied. It is found that the stream line patterns are evenly spaced to indicate the steady flow region.
- ✓ The effect of porosity parameter in the flow pattern is also discussed. It is shown that the streamlines inside the porous region are always parallel to the main flow direction regardless of the blockage ratio. If the values of porosity parameter are decreased on the steady plane flow the stream line pattern becomes negligible.

APPENDIX

1. $\beta_1 = \frac{1}{2}\left(\lambda_1 + \frac{\lambda_0}{\rho} + \frac{n}{\rho\alpha_1}\right)$
2. $\beta_2 = \frac{1}{2}\left(\lambda_1 + \frac{\lambda_0}{\rho} - \frac{m}{\rho\alpha_1}\right)$
3. $\beta_3 = \lambda_3 + \lambda_5$
4. $\beta_4 = \frac{1}{2}\left(\lambda_1 + \frac{\lambda_0}{\delta}\right)$
5. $\beta_5 = \frac{1}{2}\left(\lambda_1 - \frac{\lambda_0}{\delta}\right)$
6. $\beta_6 = \frac{n}{\gamma}$
7. $\beta_7 = \frac{m}{\gamma}$
8. $\beta_8 = \frac{1}{2}\left(\lambda_1 + \frac{\lambda_0}{\rho}\right)$
9. $\beta_9 = \frac{n_1}{\eta}$
10. $\beta_{10} = \frac{m_1}{\zeta}$
11. $\delta = \sqrt{\rho\lambda\alpha_1}$
12. $\gamma = 6\alpha_1\delta$
13. $\eta = \alpha_1^2 n_2^2 (n_2^2 + \lambda)$
14. $\zeta = \alpha_1^2 m_2^2 (m_2^2 + \lambda)$

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